Single-Sample Prophet Inequalities via Greedy-Ordered Selection

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Joint work with C.Caramanis, P.Dütting, F.Fusco, P.Lazos, S.Leonardi, O.Papadigenopoulos, E.Pountourakis, and R.Reiffenhäuser

Gambling Against a Prophet

Single-Sample Prophet Inequalities (SSPIs) are a simple variation on the classic *Prophet Inequality* problem:

- N rewards $X_i \sim \mathcal{D}_i$ arrive one at a time
- Gambler must *irrevocably* decide whether to:
  a) Collect X_i and end the game, or
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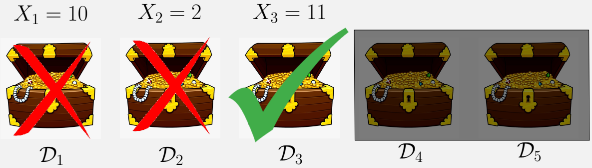

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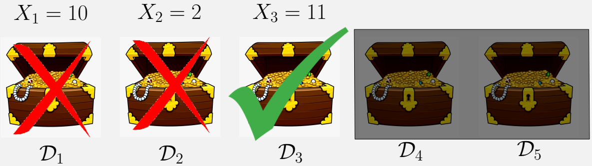

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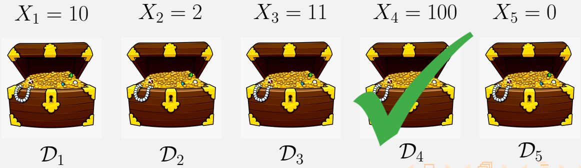

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- **Goal**: Maximize expected reward collected by Gambler, relative to that of an *all-knowing* Prophet.
  - i.e., design an "α-competitive" Gambler:

$$\inf_{\mathcal{D}=\mathcal{D}_1\times\ldots\times\mathcal{D}_N} \frac{\mathbb{E}_{\mathcal{D}}\left[\mathit{Gambler}\right]}{\mathbb{E}_{\mathcal{D}}\left[\mathit{Prophet}\right]} \geq \frac{1}{\alpha}.$$

for *smallest possible* $\alpha \geq 1$

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Notable Results:

- $\exists$ a 2-competitive *threshold-based* Gambler policy
- **No** policy can be < 2-competitive
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Question

What (if anything) can a Gambler do if she has only a **single sample** *from each* $\mathcal{D}_i$*?*

Gambling Against a Prophet with a *Single* Sample

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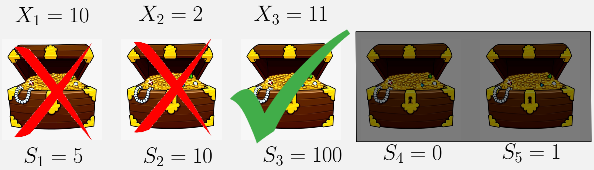

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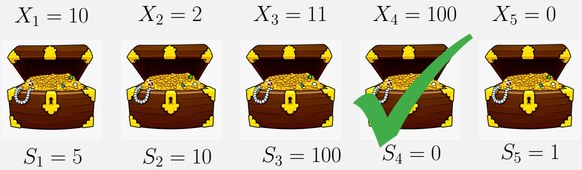

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- Prophet collects *largest* reward $\max_{i \in [N]} X_i$.
- Perhaps surprisingly, Rubenstein, Wang, and Weinberg (ITCS'20) proved that $\exists$ a 2-competitive (hence *optimal*) **single-sample** policy:
  - Accept the first reward $\geq \tau = \max_i S_i$

Beyond Single-Choice Prophet Inequalities

- Rewards can be collected subject to **combinatorial constraints**:
  - Matroid (e.g., choose k, spanning trees, ...)
  - Matchings
  - Combinatorial Auctions
- Optimal policies are known for some of these settings (e.g., matroids), but require *distributional* knowledge...

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Can we do better??

Our Contributions

1. Analyze SSPIs *beyond* single-choice **directly**, *without* reducing to OOS, via an idea we term **greedy-ordered selection**
2. Identify a common property of matroids exploited in many OOS algorithms — a *partition property* — which can be used (together with the *optimal* single-choice SSPI) to obtain improved competitive guarantees
3. Discuss some interesting new connections between SSPIs and OOSs

Our Contributions

Our focus today:

1. Analyze SSPIs *beyond* single-choice **directly**, *without* reducing to OOS, via an idea we term **greedy-ordered selection**
   - Specifically for the case of *matching with edge arrivals*
2. Identify a common property of matroids exploited in many OOS algorithms — a *partition property* — which can be used (together with the *optimal* single-choice SSPI) to obtain improved competitive guarantees
3. Discuss some interesting new connections between SSPIs and OOSs

Some Notable Results

	Combinatorial set	Previous best	Our results
Greedy-ordered selection	General matching (edge arrivals)	512	16
	Budget-additive combinatorial auction	N/A	24
	Bipartite matching (edge arrivals)	256	16
		6.75 (degree-d)	16
		$\mathcal{O}(d^2)$-samples	1 sample
	Bipartite matching (vertex arrivals)	13.5	8
	Transversal matroid	16	8
Partition property	Graphic matroid	8	4
	Co-graphic matroid	12	6
	Low density matroid	$4\gamma(M)$	$2\gamma(M)$
	Column k-sparse linear matroid	$4k$	$2k$

Table: Summary of main results

Main Idea: Greedy-Ordered Selection

Our main results (for matchings and combinatorial auctions) are obtained through a framework we call **greedy-ordered selection**. The general technique is:

1. Design a threshold-based Gambler policy
2. Couple this policy to an *equivalent* "offline" algorithm which traverses samples *and* rewards *together* in decreasing order of weight
3. Guarantee that "important" elements are "typically" selected by this offline algorithm (which is *easier to analyze*)

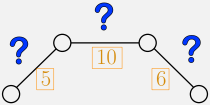
Greedily Gambling Against a Prophet with a *Single* Sample

The greedy *online* policy for matching with edge arrivals:

- Samples $S_e \sim \mathcal{D}_e$ for each edge e given, rewards $X_e \sim \mathcal{D}_e$ arrive one at a time
  - Offline, compute the greedy (maximal) matching M_S on the *samples*
  - Set a threshold τ_e to be the weight of the **heaviest edge in M_S adjacent to** e.
  - Online, accept each arriving edge e if it is feasible and $X_e \geq \tau_e$.

Matching under edge arrivals with a sample

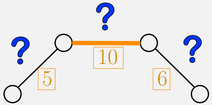
Offline (samples shown below in orange)



Matching under edge arrivals with a sample

Offline (samples shown below in orange, threshold $\tau_e = 10$ for every edge)

- Compute greedy matching on *samples* M_S



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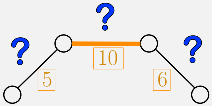
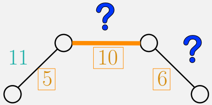


Online (rewards shown above in blue)

- Rewards X_e arrive one at a time



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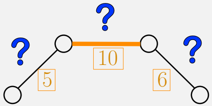

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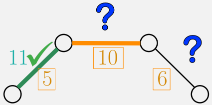
- Rewards X_e arrive one at a time
- Accept an arriving edge e iff it is:
  1. Feasible
  2. Above τ_e, the heaviest edge in M_S adjacent to e



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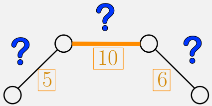

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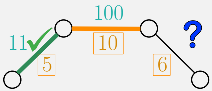

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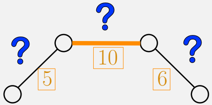

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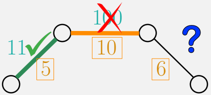

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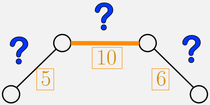

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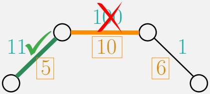

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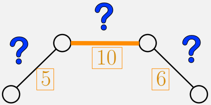

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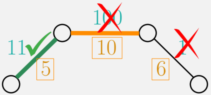

An equivalent offline simulation

The equivalent *offline* policy for matching with edge arrivals:

- Deferred decisions (offline):
  - (Conceptually) generate 2 "anonymous" values $V_{1,e}, V_{2,e} \sim \mathcal{D}_e$ for each edge e (relabel s.t. $V_{1,e} > V_{2,e}$)
  - Greedily traverse these 2n values:
    1. When $V_{1,e}$ is encountered, flip a *fair coin* to determine "status" (reward/sample)
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- Compute the greedy sample solution, M_S, and the thresholds, τ_e, *exactly* as before
- Accept elements online *exactly* as before

Matching under edge arrivals with a sample

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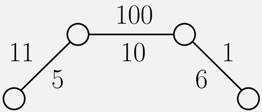

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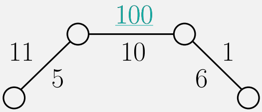

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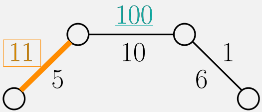

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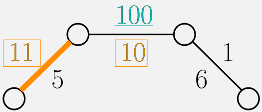

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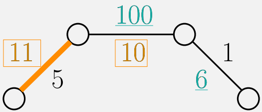

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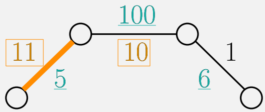

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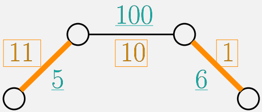

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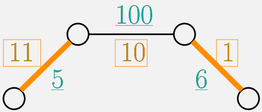

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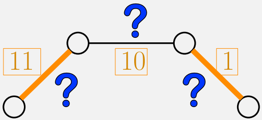

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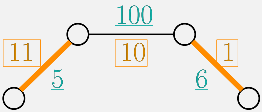

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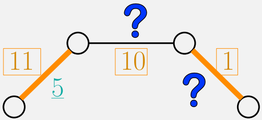

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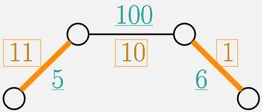

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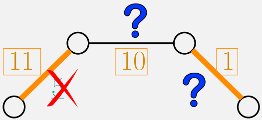

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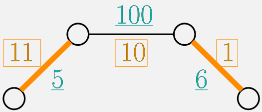
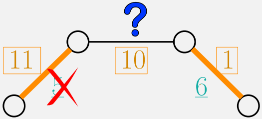


Online (rewards revealed sequentially to Gambler)



Matching under edge arrivals with a sample

Offline ($2n$ "anonymous" values below and above)

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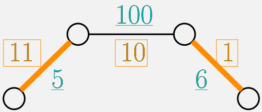

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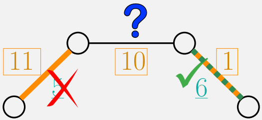

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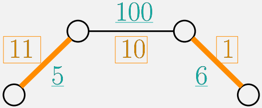

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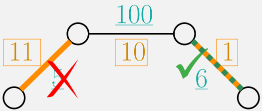

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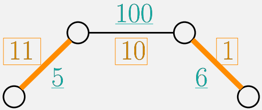

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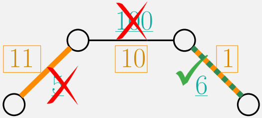


Note

- This *equivalent algorithm* viewpoint has been exploited in the secretary algorithm literature (e.g., Korula and Pal '09, Ma, Tang, and Wang '11)
- Key difficulty in our setting:
  - The status of the $V_{2,i}$'s (i.e., whether they are a sample or reward) are *correlated* with status of the corresponding $V_{1,i}$

Safe Elements

A key idea of our proofs is constructing a set of “heavy” elements which are (effectively) guaranteed to be collected, *even* in *adversarial* order

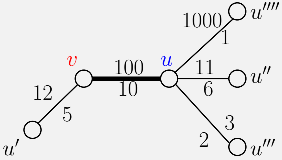
Safe Edges (Matching with Edge Arrivals)

We can an edge $e = \{v, u\}$ *"safe" for a vertex* v if:

1. $V_{1,e}$ is a **reward** that would be in the greedy solution *w.r.t. samples*, *if it were a sample*
   - "Could be in the greedy solution"
2. No edge neighboring v can block e from being accepted
   - "No conflicts with v"
3. No edge of *smaller weight* than e neighboring u can block e from being accepted
   - "No *small* conflicts with u"

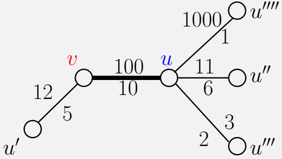
Manipulating safe edges via greedy-ordered selection

Let's see how, through **greedy-ordered selection**, to ensure that a "heavy" edge is "safe"



Manipulating safe edges via greedy-ordered selection

Rewards in blue, samples in orange. Initially, all values are **anonymous**.
Goal: make $e = \{v, u\}$ *safe for* v.



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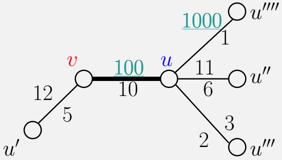

- Start with any run of the greedy offline algorithm (which determines sample/reward status of the $2n$ values) such that:
  1. $e = \{v, u\}$ could be added to the **greedy solution w.r.t. samples**, *if* $V_{1,e} = 100$ *were a sample*
  2. $V_{1,e} = 100$ is a **reward**

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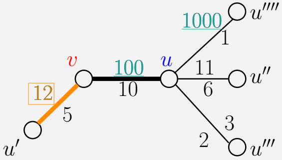

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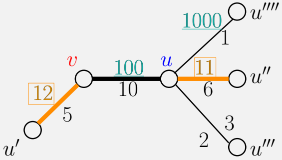

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- Fix a *small number* of coin flips to guarantee safety of e
- After fixing 2 coin flips, $e = \{v, u\}$ is safe for v (but *not* for u)

Proof for Matching with Edge Arrivals

With the safe edges defined, the competitive guarantee is (almost) immediate:

$$\mathbb{E}\left[w(\mathrm{ALG})\right] \geq \mathbb{E}\left[w(\text{Safe Edges})\right]$$

Can prove that the algorithm collects at least the weight of the safe edges

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Loss to ensure edges are *safe*

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Loss due to the greedy matching (2-approximation of OPT)

Reducing OOS to "Pointwise"-SSPIs

- Prophet inequalities appear "easier" than secretary problems:
  - $\exists$ a 2-approximation for matroid prophet inequalities
  - Best known (order-oblivious) matroid secretary is $\mathcal{O}(\log\log(\mathrm{rank}))$

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  - $\exists$ a 2-approximation for matroid prophet inequalities
  - Best known (order-oblivious) matroid secretary is $\mathcal{O}(\log\log(\mathrm{rank}))$
- What about SSPIs? Is there hope for a 2-competitive matroid SSPI?

Reducing OOS to "Pointwise"-SSPIs

Recall: Equivalent sample/reward generation for SSPIs

Viewpoint 1 (all offline):

1. Two samples $V_{1,e}, V_{2,e} \sim \mathcal{D}_e$ are drawn **independently** for every element e for *arbitrary* $\mathcal{D}_e$
2. A *single* independent coin flip for each e decides which is a sample/reward

Viewpoint 2:

1. Draw samples $S_e \sim D_e$ for each element e *offline*
2. Online, one at a time, draw reward $X_e \sim \mathcal{D}_e$

Reducing OOS to “Pointwise”-SSPIs

Definition (“Pointwise”-SSPI)

An SSPI which maintains its competitive guarantee when the rewards/samples are generated as follows:

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Observation

Every known SSPI is actually a "Pointwise"-SSPI

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An α-competitive “Pointwise”-SSPI on any downward-closed feasible set implies an 2α-competitive OOS on the same feasible set

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- *Partial* converse to the reduction of AKW’13!

A closing question

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Are there SSPIs which are **not** *"Pointwise"??*

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Thanks for listening!